MA 225 Test 1

SEPTEMBER 22, 2010

Dr. Franke Name ____________________

Show all work. You may not use a calculator.

1. (15%) Given the statement: No isosceles triangle is an equilateral triangle. (Universe is all triangles.)

   (a) translate the statement into symbols with quantifiers.

   (b) write in symbols a useful denial of your statement in (a.)

   (c) translate your answer in (b.) back into English.

2. (5%) Define what is meant by a counterexample to $(\forall x)P(x) \Rightarrow Q(x)$.

3. (8%) Outline a proof of $P \Rightarrow Q \vee R$ that uses contraposition.

4. (10%) Given the proposition: If f is continuous on $[a, b]$ and differentiable on (a, b), then there is an $x \in (a, b)$ so that

$$f'(x) = \frac{f(b) - f(a)}{b - a}.$$

   (a) Write the converse.

   (b) Write the contrapositive.

5. (8%) Find the truth table for $(P \vee \sim Q) \Rightarrow \sim (P \wedge Q)$. Determine if this proposition is a tautology, a contradiction or neither. (explain)

6. (24%)For each of the following, determine if it is true or false.

   (a) $(\exists x)(\forall y)(2x = y)$ (assume the universe is the real numbers)

   (b) $(\forall y)(\exists x)(x = \sin(y))$ (assume the universe is the real numbers)

   (c) $(\forall x)(\exists y)(xy$ is odd$)$ (assume the universe is the integers)

   (d) $(\forall y)(\exists! x)(y = x^3)$ (assume the universe is the real numbers)

   (e) If $P(x)$ is an open statement then $\sim [(\exists! x)P(x)]$ is equivalent to $[(\forall x)(\sim P(x))] \vee [(\exists y)(\exists z)[P(y) \wedge P(z) \wedge (y \neq z)]$.

   (f) $P \wedge Q \Rightarrow R$ is equivalent to $(P \Rightarrow R) \wedge (Q \Rightarrow R)$.

7. (30%) Prove each of the following:

   (a) If n is an integer, then $5n^2 + 3n$ is an even number.(Use 2 cases.)

   (b) If a, b, c and d are natural numbers, a divides c and b divides d, then ab divides cd.

1. 5 $(\forall x)$ x is isosceles $\Rightarrow$ x is not equilateral

5 $(\exists x)$ x is isosceles and x is equilateral

5 Some isosceles triangle is an equilateral triangle.

2. 5 A counterexample is an x where $P(x)$ and $\sim Q(x)$ are both true

3. 8 Assume $\sim Q$ and $\sim R$

$\vdots$

Thus $\sim P$.

Hence $\sim Q \wedge \sim R \Rightarrow \sim P$

Thus by contraposition $P \Rightarrow Q \vee R$.

4. 5 a.) If there is an $x \in (a,b)$ so that $f'(x) = \dfrac{f(b)-f(a)}{b-a}$

then f is continuous on $[a,b]$ and differentiable on (a,b).

b.) 5 If $\forall x \in (a,b)$ $f'(x) \neq \dfrac{f(b)-f(a)}{b-a}$ then f is not continuous

on $[a,b]$ or f is not differentiable on (a,b).

5.

P	Q	$\sim Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$P \vee \sim Q$	$(P \vee \sim Q) \Rightarrow \sim(P \wedge Q)$
T	T	F	T	F	T	F
F	T	F	F	T	F	T
T	F	T	F	T	T	T
F	F	T	F	T	T	T

not all T so not a tautology

Not all F so not a contradiction

6

a F

b T

c F

d T

e T

f F

7a.) Let n be an integer. Either n is even or n is odd.

Assume n is even. Then $\exists\, k \in \mathbb{Z} \ni n = 2k$.

$$5n^2+3n = 5(2k)^2 + 3(2k) = 2(10k^2+3k).$$

$10k^2+3k \in \mathbb{Z}$ by closure. Thus $5n^2+3n$ is even.

Thus if n is even then $5n^2+3n$ is even.

Assume n is odd. Then $\exists\, \ell \in \mathbb{Z} \ni n = 2\ell+1$.

$$5n^2+3n = 5(2\ell+1)^2 + 3(2\ell+1) = 5(4\ell^2+4\ell+1) + 3(2\ell+1)$$

$$= 20\ell^2 + 20\ell + 5 + 6\ell + 3 = 20\ell^2 + 26\ell + 8$$

$$= 2(10\ell^2+13\ell+4).$$ $10\ell^2+13\ell+4 \in \mathbb{Z}$ by closure.

Thus $5n^2+3n$ is even.

Hence if n is odd, $5n^2+3n$ is even.

Since we have check both case n even and n odd, if n is an integer then $5n^2+3n$ is even.

1. b. Let a, b, c, d be natural numbers. Assume a divides c and b divides d. Then $\exists\, k \in \mathbb{N}$ and $l \in \mathbb{N} \ni$

$c = ak$ and $d = bl$. Thus

$cd = (ak)(bl) = ab(kl)$. $kl \in \mathbb{N}$ by closure.

Thus ab divides cd.

Therefore if a, b, c, d are natural numbers, a divides c and b divides d, then ab divides cd.