

MA 225 Test 1

SEPTEMBER 22, 2010

Dr. Franke

Name _____

Show all work. You may not use a calculator.

- (15%) Given the statement: No isosceles triangle is an equilateral triangle. (Universe is all triangles.)
 - translate the statement into symbols with quantifiers.
 - write in symbols a useful denial of your statement in (a.)
 - translate your answer in (b.) back into English.
- (5%) Define what is meant by a counterexample to $(\forall x)P(x) \Rightarrow Q(x)$.
- (8%) Outline a proof of $P \Rightarrow Q \vee R$ that uses contraposition.
- (10%) Given the proposition: If f is continuous on $[a, b]$ and differentiable on (a, b) , then there is an $x \in (a, b)$ so that

$$f'(x) = \frac{f(b) - f(a)}{b - a}.$$

- Write the converse.
 - Write the contrapositive.
- (8%) Find the truth table for $(P \vee \sim Q) \Rightarrow \sim (P \wedge Q)$. Determine if this proposition is a tautology, a contradiction or neither. (explain)
 - (24%) For each of the following, determine if it is true or false.
 - $(\exists x)(\forall y)(2x = y)$ (assume the universe is the real numbers)
 - $(\forall y)(\exists x)(x = \sin(y))$ (assume the universe is the real numbers)
 - $(\forall x)(\exists y)(xy \text{ is odd})$ (assume the universe is the integers)
 - $(\forall y)(\exists! x)(y = x^3)$ (assume the universe is the real numbers)
 - If $P(x)$ is an open statement then $\sim [(\exists! x)P(x)]$ is equivalent to $[(\forall x)(\sim P(x))] \vee [(\exists y)(\exists z)[P(y) \wedge P(z) \wedge (y \neq z)]]$.
 - $P \wedge Q \Rightarrow R$ is equivalent to $(P \Rightarrow R) \wedge (Q \Rightarrow R)$.
 - (30%) Prove each of the following:
 - If n is an integer, then $5n^2 + 3n$ is an even number. (Use 2 cases.)
 - If a, b, c and d are natural numbers, a divides c and b divides d , then ab divides cd .

1. $\frac{5}{5} (\forall x)$ x is isosceles $\Rightarrow x$ is not equilateral

$\frac{5}{5} (\exists x)$ x is isosceles and x is equilateral

$\frac{5}{5}$ Some isosceles triangle is an equilateral triangle.

2. $\frac{5}{5}$ A counterexample is an x where $P(x)$ and $\sim Q(x)$ are both true

3. Assume $\sim Q$ and $\sim R$

$\frac{5}{5}$ \vdots

Thus $\sim P$.

Hence $\sim Q \wedge \sim R \Rightarrow \sim P$

Thus by contraposition $P \Rightarrow Q \vee R$.

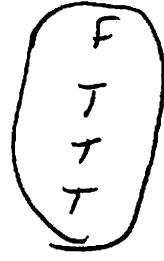
4. a.) $\frac{5}{5}$ If there is an $x \in (a, b)$ so that $f'(x) = \frac{f(b) - f(a)}{b - a}$

then f is continuous on $[a, b]$ and differentiable on (a, b) .

b.) $\frac{5}{5}$ If $\forall x \in (a, b)$ $f'(x) \neq \frac{f(b) - f(a)}{b - a}$ then f is not continuous on $[a, b]$ or f is not differentiable on (a, b) .

5.

P	Q	$\sim Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$P \vee \sim Q$	$(P \vee \sim Q) \leftrightarrow \sim(P \wedge Q)$
T	T	F	T	F	T	F
F	T	F	F	T	F	T
T	F	T	F	T	T	T
F	F	T	F	T	T	T



not all T so not a tautology

not all F so not a contradiction

6

- a F
- b T
- c F
- d T
- e T
- f F

7a.) Let n be an integer. Either n is even or n is odd.

[assume n is even. Then $\exists k \in \mathbb{Z} \ni n = 2k$,

$$5n^2 + 3n = 5(2k)^2 + 3(2k) = 2(10k^2 + 3k).$$

$10k^2 + 3k \in \mathbb{Z}$ by closure. Thus $5n^2 + 3n$ is even.

Thus if n is even then $5n^2 + 3n$ is even.

[assume n is odd. Then $\exists l \in \mathbb{Z} \ni n = 2l + 1$,

$$5n^2 + 3n = 5(2l+1)^2 + 3(2l+1) = 5(4l^2 + 4l + 1) + 3(2l+1)$$

$$= 20l^2 + 20l + 5 + 6l + 3 = 20l^2 + 26l + 8$$

$$= 2(10l^2 + 13l + 4). \quad \underline{10l^2 + 13l + 4 \in \mathbb{Z} \text{ by closure.}}$$

Thus $5n^2 + 3n$ is even.

Hence if n is odd, $5n^2 + 3n$ is even.

[since we have checked both cases n even and n odd,
if n is an integer then $5n^2 + 3n$ is even.]

7. b. Let a, b, c, d be natural numbers. Assume a divides c
and b divides d . Then $\exists k \in \mathbb{N}$ and $l \in \mathbb{N}$ \Rightarrow

$$\underline{c = ak}, \text{ and } \underline{d = bl}. \text{ Thus}$$

$$cd = (ak)(bl) = ab(kl). \quad \underline{kl \in \mathbb{N} \text{ by closure.}}$$

Thus ab divides cd .

[Therefore if a, b, c, d are natural numbers, a divides c
and b divides d , then ab divides cd .